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ASTROMETRIC RESOLUTION OF SEVERELY DEGENERATE BINARY MICROLENSING EVENTS

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ABSTRACT

We investigate whether the "close/wide" class of degeneracies in caustic-crossing binary microlensing events can be broken astrometrically. Dominik showed that these degeneracies are particularly severe because they arise from a degeneracy in the lens equation itself rather than a mere "accidental" mimicking of one light curve by another. A massive observing campaign of five microlensing collaborations was unable to break this degeneracy photometrically in the case of the binary lensing event MACHO 98-SMC-1. We show that this degeneracy indeed causes the image centroids of the wide and close solutions to follow an extremely similar pattern of motion during the time when the source is in or near the caustic. Nevertheless, the two image centroids are displaced from one another, and this displacement is detectable by observing the event at late times. Photometric degeneracies therefore can be resolved astrometrically, even for these most severe cases.

Subject headings: astrometry — Galaxy: stellar content — gravitational lensing — Magellanic Clouds

1. INTRODUCTION

Caustic-crossing binary microlensing events are potentially very useful, but their interpretation can be problematic. If there is good photometric coverage of a caustic crossing, one can measure the limb darkening of the source (Albrow et al. 1999b; Afonso et al. 2000; Albrow et al. 2000), and if this is combined with spectroscopy, one can resolve the source's spectral features as a function of angular position (Gaudi & Gould 1999). If there is sufficiently good coverage of the event to obtain a *unique* binary solution, then one determines the binary mass ratio and in some cases other information about the binary (Albrow et al. 2000). If this information can be obtained for a number of events, then one can infer statistical properties about the binaries in the lens population as a whole (Gaudi & Sackett 2000).

In some cases, it is possible to measure the proper motion μ of the lens relative to the observer-source line of sight. This requires three pieces of information from the photometric light curve. First, one must measure the time it takes the source star to cross the caustic, Δt, which can be done from photometry of the caustic-crossing alone (e.g., Albrow et al. 1999a, 1999c). Second, one must measure the angle ϕ of this crossing, which requires a *unique* binary solution for the event as a whole. Third, one must determine the angular size θ_* of the source from its color and apparent magnitude using an empirically calibrated relation (van Belle 1999). The color is quite easily measured, but the apparent magnitude again requires a *unique* binary solution. The proper motion is then $\mu = \theta_*/(\Delta t\,|\sin\phi|)$. Five groups (Afonso et al. 1998; Udalski et al. 1998; Alcock et al. 1999; Albrow et al. 1999a; Rhie et al. 1999) combined observations from eight observatories to measure the proper motion of MACHO 98-SMC-1 and so proved beyond reasonable doubt that the lens was in the SMC and not in the Galactic halo (Afonso et al. 2000).

As can be seen from this brief summary, many applications of binary lenses require that one obtain a unique binary-lens solution to the event. However, Dominik (1999a) presented multiple solutions to a number of previously published events and argued that degeneracies of this sort may be generic.

Han, Chun, & Chang (1999) therefore investigated whether such degeneracies can be broken astrometrically. In a binary lensing event there are three or five images, depending on whether the source is outside or inside the caustic. The combined light of these three or five images makes up the photometric light curve, which is the only effect that has been observed to date. The images are separated by of order the Einstein radius, $\theta_{\rm E}$, which is a few hundred microarcseconds for typical events. Hence the images cannot be separately resolved with any existing or planned instrument. However, the image centroid deviates from the source position by a vector amount $\delta\boldsymbol{\theta}_c = (\delta\theta_{c,x}, \delta\theta_{c,y})$, which is also of order $\theta_{\rm E}$. The *Space Interferometry Mission* (*SIM*) with its planned $\sim$4 μas precision will therefore have the capability to measure this deviation, and several ground-based interferometers may also achieve the necessary precision.

Han et al. (1999) explicitly showed that the four solutions that Dominik (1999a) presented for OGLE 7 (Udalski et al. 1994), which all fit the observed light curve extremely well, had radically different astrometric trajectories. Hence, had there been astrometric data, this degeneracy could have easily been broken.

Subsequently, Albrow et al. (1999c) developed a general method for finding solutions in events with well-covered caustic crossings, and Afonso et al. (2000) applied this method to MACHO 98-SMC-1 and found two solutions that fit the full data set equally well. See Figures 1 and 2 below. In spite of Dominik's (1999a) work showing that degeneracies in earlier light curves were common, the degeneracy in MACHO 98-SMC-1 came as something of a surprise because the five-collaboration data set was far superior to those of the events investigated by Dominik (1999a).

However, simultaneously with Afonso et al.'s (2000) *empirical* discovery of a severe degeneracy in MACHO 98-SMC-1, Dominik (1999b) found *an entire class* of severe degeneracies between "close" and "wide" binaries, i.e., binaries with projected angular separations small and large compared to θ_E. Indeed, MACHO 98-SMC-1 turns out to be a particular case of this class.

The disturbing thing about the Dominik (1999b) close/wide degeneracies, and what makes them so difficult to break, is that they derive from a degeneracy in the lens equation itself. That is, while some of the degeneracies found by Dominik (1999a) may be regarded as due to "accidental" similarities between different light curves (the sum of the magnification of the three or five images), the close/wide degeneracies are rooted in the similarities of the individual images. This immediately raises the question of whether it is possible to break these degeneracies at all, even using astrometric data as Han et al. (1999) showed could be done for the earlier (Dominik 1999a) degeneracies. We address that question here.

2. ASTROMETRIC RESOLUTION

To investigate this question, we examine the astrometric behavior of the two solutions[1] to MACHO 98-SMC-1 found by Afonso et al. (2000). The Einstein radii θ_E are 74 and 167 μas, the Einstein crossing times t_E are 99 and 165 days, the mass ratios M_2/M_1 are 0.50 and 4.17, and the separations are $d\theta_E$, where $d = 0.54$ and 3.25, respectively. Here M_1 is the mass of the component that is closer to the caustic that the source passes through. The full solutions are described in Tables 1 and 2 of Afonso et al. (2000).

Figure 1 shows the trajectory of the source relative to the binary in the two solutions. Figure 2 is adapted from Figures 3 and 4 of Afonso et al. (2000) and shows the predicted light curves in the I band for these two solutions. Time is shown as $HJD' = HJD - 2{,}450{,}000$. The data are binned in 1 day intervals except in the immediate neighborhood of the caustics, where there are 0.1 day bins. Data taken in other bands are adjusted to the I-band system using the source and background fluxes from each observatory and each band as determined from the overall fit. See Afonso et al. (2000). The main conclusion from Figure 2 is that the two solutions are essentially identical from a photometric standpoint.

[1] Afonso et al. (2000) actually found two wide solutions, a static and a rotating one. The static solution was completely consistent with the data in the neighborhood of the observable event, but was by chance ruled out by early data about 500 days before the event. For simplicity, and because we are trying to illustrate a general principle rather than specifically investigating MACHO 98-SMC-1, we will use the static wide solution and therefore will ignore the early data. This will allow us to compare two static systems, thus ensuring that differences between the astrometric trajectories are not due to the fact that one is rotating and the other is not.

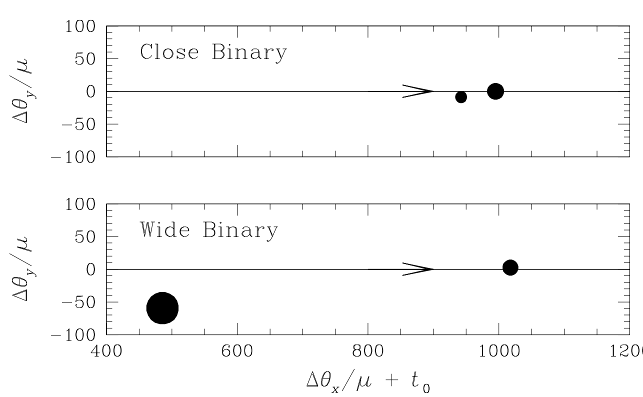

FIG. 1.—Positions of the components of the binary lens MACHO 98-SMC-1 in the two models of Afonso et al. (2000). The size (area) of the dots indicates the relative masses of the components. The panels show angular position scaled by the proper motion μ. The units are therefore time, which means that the source trajectory is shown as a function of $\Delta\theta_x/\mu + t_0 = HJD'$, and are therefore the same in the two panels.

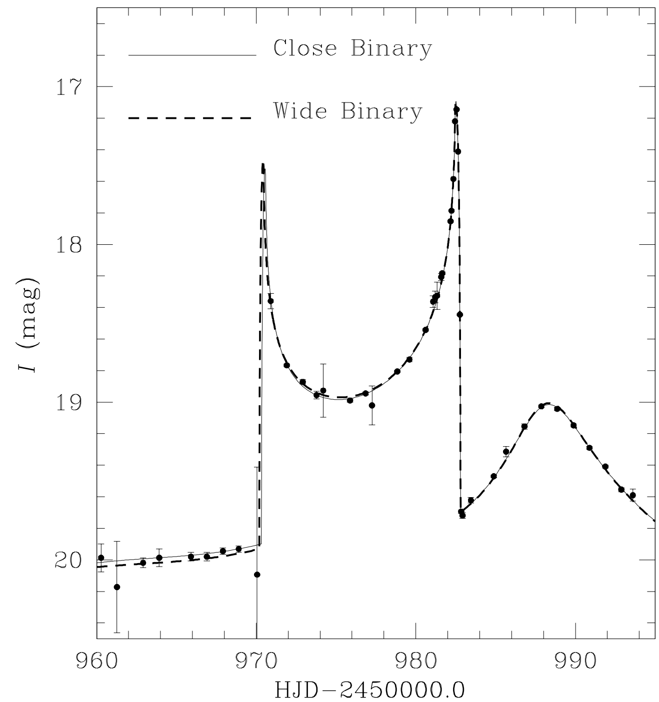

FIG. 2.—Light curves for close binary (*solid line*) and wide binary (*dashed line*) models for the caustic-crossing binary microlensing event MACHO 98-SMC-1, together with binned data from five microlensing collaborations. Both the curves and the data are adapted from Figs. 3 and 4 of Afonso et al. (2000). The non-I points have been put on the I-band system using the solutions of Afonso et al. (2000), so that both curves could be shown on the same plot. The two models predict virtually identical photometric results.

Figure 3 shows the astrometric deviation of the light centroid from the source position for the close and wide solutions, respectively. The dashed portions of the curve show the jumps at the times of the caustic crossings. These jumps would be discontinuous for a point source, but in fact take place by a (rapid) continuous motion for a finite source. Note that the two displacement curves look extremely similar. This similarity derives from the underlying degeneracy in the lens equation that was discovered by Dominik (1999b).

Although the *pattern* of centroid motion is extremely similar in the two cases, the two curves are actually displaced from one another by an offset

$$\Delta\boldsymbol{\delta\theta}_c(t) = \boldsymbol{\delta\theta}_{c,\mathrm{close}}(t) - \boldsymbol{\delta\theta}_{c,\mathrm{wide}}(t)\ , \tag{1}$$

which is about 40 μas, i.e., $\sim 0.5\theta_E$ for the close binary, or by $\sim 0.25\theta_E$ for the wide binary. Such an offset is not observable if the observations are restricted to times when $\Delta\boldsymbol{\delta\theta}_c$ is approximately constant. However, at very early or very late times, $\boldsymbol{\delta\theta}_c \to 0$, since when the source is far from the lens, the

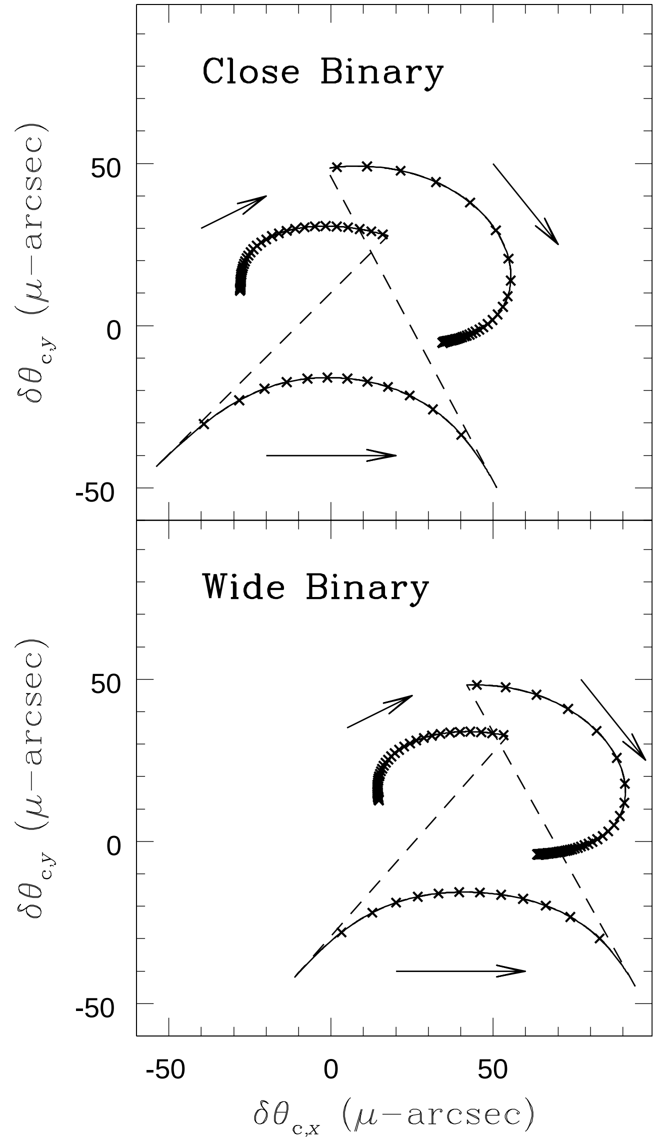

FIG. 3.—Astrometric deviation $\delta\boldsymbol{\theta}_c$ of the image centroid from the source position for the same two models shown in Fig. 2 and over the same time interval, $960 \leq \mathrm{HJD}' \leq 995$. The crosses show the progress of the event in 1 day intervals, and the arrows designate the direction of the centroid motion. The dashed lines show the "instantaneous jumps" that the image centroid of a point source would undergo at the caustic crossing. Finite source effects (not shown) would make these transitions continuous and would foreshorten them by about 3%. The arcs at the bottoms represent the image centroid positions when the source is inside the caustic. The pattern of motion in the two cases looks extremely similar, confirming the photometric degeneracy illustrated in Fig. 2. However, the two trajectories are offset by ~ 40 μas, meaning that they can be distinguished if the zero point of astrometry is established by sufficiently late-time observations.

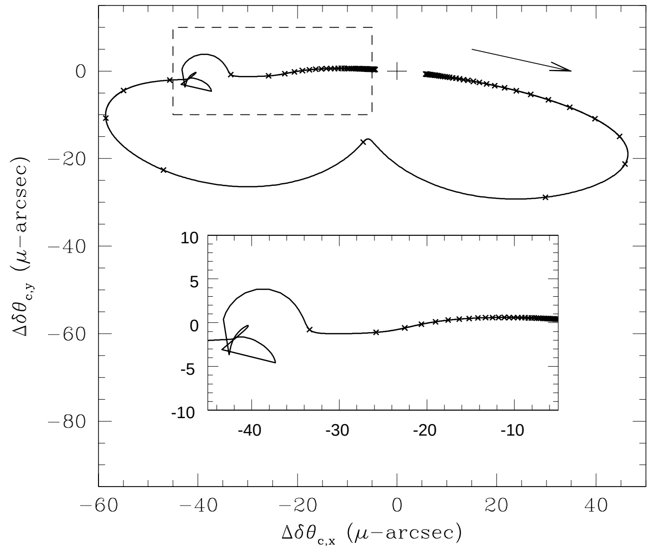

FIG. 4.—Difference $\Delta\delta\boldsymbol{\theta}_c$ between the two astrometric deviations shown in Fig. 3, with evaluations every 100 days shown by crosses. The full figure shows $\Delta\delta\boldsymbol{\theta}_c$ over a period of 20 years, while the inset shows only the period during and after the caustic crossing (when astrometric measurements might reasonably have been triggered). During the year after the caustic crossing, $\Delta\delta\boldsymbol{\theta}_c$ changes by ~ 20 μas. To the extent that this is consistent with uniform motion, it could not be disentangled from the uniform proper motion of the source. However, during the subsequent year, $\Delta\delta\boldsymbol{\theta}_c$ slows down substantially, so that its motion over 2 years could easily be distinguished from uniform motion. Thus, astrometric measurements would distinguish between the two solutions.

image and source positions coincide. Because this occurs for both models, $\Delta\delta\boldsymbol{\theta}_c$ must vanish at these times. How long, in practice, must one wait to tell the difference between the two models?

This question is addressed in Figure 4, where we plot the offset between the two models, $\Delta\delta\boldsymbol{\theta}_c(t)$, as a function of time. The main figure shows the behavior of $\Delta\delta\boldsymbol{\theta}_c(t)$ over the whole event, while the inset is restricted to times during and after the caustic crossing (when in practice astrometric observations might first be triggered). The offset shows some structure on scales of ~ 5 μas during the time when the source is inside the caustic, but the main thing to notice is that over the next year it changes by 20 μas and thus would be noticed if the event were monitored with *SIM*-like precision. The full 40 μas change would take place only after about a decade. Note that to the extent that $d\Delta\delta\boldsymbol{\theta}_c/dt$ can be approximated as a constant, $\Delta\delta\boldsymbol{\theta}_c$ cannot be detected at all, because such uniform motion can be subsumed in the fit for the proper motion of the source. However, from Figure 4, $\Delta\delta\boldsymbol{\theta}_c(t)$ slows down dramatically after about 1 year, so that after 2 years, its nonuniform motion could be unambiguously distinguished from uniform source motion.

It is also instructive to look at the behavior of Figure 4 at early times. Of course no astrometric measurements could have been taken then because there had been no signature of an event. However, the entire event could just as well have taken place in reverse. In this case *postcaustic* astrometric measurements would probe dramatic changes in the offset, much larger than the 40 μas changes in the event proceeding in its actual direction. The reason for this can be seen in Figure 1: the source passes relatively close to the companion binary member, and this passage induces a large astrometric deviation. (Indeed, this passage is so close that it induces a noticeable deviation in the photometric light curve, which is why the early data for this event ruled out the static wide solution.) In general, the source is not likely to pass close to both members, so that deviations of the type seen in Figure 4 are unlikely.

However, the ~ 40 μas offset seen in Figure 3 at times when the source is inside the caustic is a generic feature of this caustic and does not depend in any way on the direction of the source trajectory through the caustic. Therefore, it is generically possible to break the close/wide (Dominik

1999b) degeneracy astrometrically, even when it is extremely difficult to do so photometrically.

3. DISCUSSION

As a practical matter, *SIM* could not resolve the degeneracy in MACHO 98-SMC-1 because the source is only $I \sim 22$ mag, far too faint for *SIM* to follow. Most events that *SIM* could monitor would be in the bulge, where there are far more events and where the sources are much brighter. For these events, the typical Einstein radius is probably $\theta_E \sim 300$ μas, so the astrometric deviations would be several times larger than for MACHO 98-SMC-1. Hence, it seems likely that for sources that could be monitored astrometrically at all, breaking the degeneracy would be well within *SIM*'s capabilities.

Finally, we ask: What is the fundamental physical reason that the photometric degeneracy is reproduced as an astrometric degeneracy in the neighborhood of the caustic, but can be broken astrometrically at late times? This can most easily be seen by looking at Figure 1. In each model, the caustic that is crossed is associated with the mass at the right. The Einstein crossing times associated with these masses, $t'_E = [M_1/(M_1 + M_2)]^{1/2} t_E$, are about the same in the two cases, $t'_{E,\,\mathrm{close}} = 81$ days and $t'_{E,\,\mathrm{wide}} = 72$ days, respectively. We therefore show the size of mass M_1 to be $t'_{E,\,\mathrm{close}}/t'_{E,\,\mathrm{wide}} = 1.25$ times larger in the close-binary panel than in the wide-binary panel. The lens equation is very similar in the neighborhood of this mass, which is the origin of both the photometric and astrometric degeneracy. However, for the wide-binary solution, the very large mass of the companion at the left displaces the entire image structure to the right by $\sim [M_2/(M_1 + M_2)]^{1/2} \theta_E/d \sim 57$ μas. This displacement only gradually returns to zero: even at time $dt_E \sim 540$ days after the event, it has only fallen by half. By contrast, once the source has left the vicinity of M_1 of the close binary, there are no large and distant masses that could significantly displace the images relative to the source.

In the wide/close degeneracy, the wide-binary solution generically has a large-mass companion lying several Einstein radii from the position of the caustic that is crossed by the source. Hence, we expect that at the end of the caustic crossing, the image centroid will generally be substantially displaced from the source in the wide-binary solution, but that this will not be so for the close-binary solution. Therefore, it should usually be possible to break the close/wide degeneracy by tracking the image centroid at late times, just as we have shown is possible for MACHO 98-SMC-1.

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